Random quantum Ising chains with competing interactions

David Carpentier, Pierre Pujol, Kay-Uwe Giering

Random quantum Ising chains with competing interactions

David Carpentier and Pierre Pujol
Laboratoire de Physique de l'Ecole Normale Supérieure de Lyon, 46, Allée d'Italie, 69007 Lyon, France

Kay-Uwe Giering
Laboratoire de Physique de l'Ecole Normale Supérieure de Lyon, 46, Allée d'Italie, 69007 Lyon, France and
Fakultät für Physik und Geowissenschaften, Universität Leipzig, Augustusplatz 10, 04109 Leipzig, Germany
(Dated: 22 Juin 2005)

In this paper we discuss the criticality of a quantum Ising spin chain with competing random ferromagnetic and antiferromagnetic couplings. Quantum fluctuations are introduced *via* random local transverse fields. First we consider the chain with couplings between first and second neighbors only and then generalize the study to a quantum analog of the Viana-Bray model, defined on a small world random lattice. We use the Dasgupta-Ma decimation technique, both analytically and numerically, and focus on the scaling of the lattice topology, whose determination is necessary to define any infinite disorder transition beyond the chain. In the first case, at the transition the model renormalizes towards the chain, with the infinite disorder fixed point described by Fisher. This corresponds to the irrelevance of the competition induced by the second neighbors couplings. As opposed to this case, this infinite disorder transition is found to be unstable towards the introduction of an arbitrary small density of long range couplings in the small world models.

Quantum fluctuations play a crucial role in the spin glass phases of the Sr-doped cuprate La_2CuO_4 [1], or the dipolar glass $LiHo_xY_{1-x}F_4$ in a transverse field [2]. Randomly coupled quantum two level systems also appear in the understanding of the dielectric response of low temperature amorphous solids [3], and as the main low frequency source of decoherence of solid state quantum bits [4]. In all cases, the quantum fluctuations compete with the random couplings between the spin, and tend to disorder the corresponding random ordered phases.

One of the simplest random quantum model to study this competition is probably the random Ising spin model in a transverse magnetic field.

$$H = -\sum_{i,j} J_{ij}\sigma_i^z\sigma_{i+1}^z - \sum_i h_i\sigma_i^x, \qquad (1)$$

where σ_i^x, σ_i^z are the usual Pauli matrices, and the transverse fields h_i are responsible for the quantum tunneling fluctuations between the up and down states of the Ising spins. In a pioneering work, Fisher has given asymptotically exact results for this random quantum Ising model with first neighbors random ferromagnetic bonds in one dimension [5]. By using a decimation technique developed by Dasgupta and Ma [6], he described the *infinite disorder* quantum phase transition of this model. Some of the main features of this peculiar transition were a diverging dynamical exponent z, and very strong inhomogeneities manifesting through drastically different behavior between average and typical correlation functions.

Natural extensions of these results to higher dimensions have proved to be difficult. In particular, an analytical implementation of the Ma-Dasgupta decimation beyond the simple chain is extremely cumbersome. The reason is that any initial lattice except the chain is quickly randomized by the decimation. Thus one has to resort to a numerical implementation of this decimation [7, 8]. For two-dimensional regular lattices, the results for the random ferromagnetic Ising model are consistent with the survival of an infinite disorder quantum phase transition, albeit with exponents different from the one-dimensional case [7]. On the other hand, the quantum Ising spin glass, corresponding to the model (1) with both ferromagnetic (positive) and antiferromagnetic (negative) couplings, was also studied in two and three dimensions *via* Monte-Carlo simulations [9]. The numerical works found no sign of an infinite disorder quantum critical point. Since the results for the random ferromagnetic model are expected to extends to the quantum Ising spin glass, this discrepancy certainly deserves further work.

In this perspective, we investigate in this paper the stability of infinite disorder fixed point of the quantum Ising spin glass chain with respect to competing further neighbors couplings in two extreme cases. In a first step, we focus in the case where second neighbors couplings are present in model (1) besides the first neighbors couplings. The couplings are taken as either ferromagnetic or antiferromagnetic. By combining analytical (for small second neighbors couplings) and numerical decimation techniques we investigate the relevance of the presence of higher range couplings and their interplay with random signs in the couplings. We pay a special attention to the topology of the renormalized lattice, which appears crucial in the precise characterization of infinite disorder transitions.

A natural complement to this first case consists in considering this quantum Ising spin glass on a random network, obtained by adding a finite density of long range couplings between the chain's sites. Indeed, our model can be considered as a quantum analog of the classical spin models of Viana and Bray[10], although we keep a local regular topology besides the random long range couplings in our *small world* lattice[11]. In the case of classical spin glasses, these random lattice models are a natural extrapolation between the short-range model and its mean-field version. They undergo a finite temperature transition of the mean-field type, albeit with peculiarities induced by the finite connectivity[12]. Wether the infinite disorder physics survives to this tendency towards mean-field like physics is the natural question we will consider.

In the first part of this letter, we consider the model (1) on a chain, with only first and second neighbors couplings (Zig-Zag ladder) [13]. Both the first and second neighbors couplings $J^{(1)}_{i,i+1}, J^{(2)}_{i,i+2}$ can be antiferromagnetic (< 0) with probability p, and ferromagnetic with probability $1-p$. The $|J^{(1)}_{i,i+1}|$ are uniformly distributed between 0 and 1, the $|J^{(2)}_{i,i+2}|$ between 0 and $J^{(2)}_{max}$, and the transverse fields h_i between 0 and h_{max}. Note that *via* an appropriate unitary transformation we can map this system onto one where only the second neighbors couplings can have both signs, but at the cost of a modification of the magnetic properties of the system. Hence for clarity, we prefer to consider only the more natural choice defined above.

We will analyze the low temperatures behavior of this system by means of the Dasgupta-Ma decimation technique [6] which was exploited by Fisher [5] in the case, among others, of the random ferromagnetic quantum Ising chain. Its extension to the present case of mixed coupling (anti-ferromagnetic and ferromagnetic) contains one supplementary rule as detailed below. The running energy scale Ω is defined as the maximum of the amplitudes of bonds $|J_{ij}|$ and fields h_i. At each decimation step, if this maximum corresponds to a field h_i, the corresponding spin is frozen in the x direction, generating new couplings $\tilde{J}_{jk} = J_{jk} + (J_{ij}J_{ik}/\Omega)$ between all pairs (j,k) previously connected with the spin i. On the other hand, if the maximum is a ferromagnetic coupling J_{ij}, the two spins i and j are paired to form a new cluster $[ij]$ of magnetization $\mu_{[ij]} = \mu_i + \mu_j$ (where μ_i corresponds to the magnetization of cluster i), and coupling with site k $J_{[ij]k} = J_{ik} + J_{jk}$ [5]. The new rule occurs when this maximum coupling is anti-ferromagnetic. In this case, if *e.g* the magnetization μ_i is larger than μ_j, then the new cluster's magnetization reads $\mu_{[ij]} = \mu_i - \mu_j$, and the interaction with a third spin k is $J_{[ij]k} = J_{ik} - J_{jk}$. In both cases, the effective transverse field acting on the new cluster is $h_{[ij]} = h_i h_j/\Omega$.

An analytical study of the scaling behavior of the model (1) under the above decimation rules is difficult even on the Zig-Zag ladder we consider. As mentioned in the introduction, couplings J_{ij} are quickly generated on many length scales $|i-j|$, and the initial lattice is quickly randomized (see below). To fix the notation and clarify the procedure, it is useful to start by considering the evolution under the RG of the first neighbors chain, extending the result of Ref. 5 to the presence of anti-ferromagnetic couplings. We introduce the convenient logarithmic variables $\beta_i = \ln(\Omega/h_i)$, $\zeta_{i,i+1} = \ln(\Omega/|J_{i,i+1}|)$ and scaling parameter $\Gamma := \ln(\Omega_0/\Omega)$ where Ω_0 is the initial value of Ω. Their "distributions" densities are defined as $\mathcal{R}(\beta,\Gamma)$ for the fields, $\mathcal{P}^{(1+)}(\zeta,\Gamma)$ for the ferromagnetic bonds, and $\mathcal{P}^{(1-)}(\zeta,\Gamma)$ for the anti-ferromagnetic bonds. Note that while $\mathcal{R}(\beta,\Gamma)$ is normalized, for the bonds only the sum $\mathcal{P}^{(1)}(\zeta,\Gamma) = \mathcal{P}^{(1+)}(\zeta,\Gamma) + \mathcal{P}^{(1-)}(\zeta,\Gamma)$ has a norm one. As can be deduced by a gauge transformation of (1), $\mathcal{R}(\beta,\Gamma)$ and $\mathcal{P}^{(1)}(\zeta,\Gamma)$ satisfy the same differential scaling equations than in the ferromagnetic case [5] provided we use the maximum instead of the sum in the above decimation rules, which is valid for broad enough distributions. Finally, the function $\mathcal{D}(\beta,\Gamma) = \mathcal{P}^{(1+)}(\zeta,\Gamma) - \mathcal{P}^{(1-)}(\zeta,\Gamma)$ is found to satisfy the same scaling equation than $\mathcal{P}^{(1)}$. The fixed point $\mathcal{R} = \mathcal{P}^{(1)} = \mathcal{P}^*(x,\Gamma) = e^{-x/\Gamma}/\Gamma$ of the ferromagnetic chain[5] is easily extended to the two following case : the above ferromagnetic point now corresponds to the solution $\mathcal{D} = \mathcal{P}^*$, or $\mathcal{P}^{(1+)} = \mathcal{P}^{(1)} = \mathcal{P}^*$, $\mathcal{P}^{(1-)} = 0$. As expected, it can be explicitely checked in the RG equations that this fixed point is unstable towards the proliferation of anti-ferromagnetic bonds. The new transition point corresponds to the solution $\mathcal{D} = 0$, or $\mathcal{P}^{(1+)} = \mathcal{P}^{(1-)} = \mathcal{P}^*/2$, corresponding to an equal density of random positive and negative couplings. Hence, we will loosely call it the spin glass fixed point by analogy with the physics of the classical model in higher dimensions. The characteristics of the transtion from the the ferromagnetic to the disordered phase obtained by Fisher [5] translate to the present spin-glass fixed point into an average linear susceptibility (under the application of a small z field $\tilde{h}$) which diverges as $\chi(T) \sim |\ln T|^{\phi-2}/T$ where $\phi = (1+\sqrt{5})/2$. Similarly, we extract the scaling behaviour of the average non linear susceptibility $\chi_{nl}(T) = \left[\frac{\partial^3}{\partial \tilde{h}^3}\Big|_{\tilde{h}=0} < M > (\tilde{h}) \right]$, where $[\dots]$ denotes an ensemble average, as $\chi_{nl}(T) \sim |\ln T|^{2\phi-2}/T^3$.

Having clearly defined the notation and fixed points for the chain, we can now study perturbatively their stability with respect to small second neighbors competing interactions. To first order, such an analysis can be conducted by considering the presence of $J^{(2)}$ negligible compared to the $J^{(1)}$, and checking whether this condition is self-consistently preserved under the re-scaling. More precisely, we will assume that *(i)* a $J^{(2)}_{i,i+2}$ will never constitute the highest energy in the system and therefore never be decimated *(ii)* in sums, the $J^{(2)}_{i,i+2}$ are negligible with respect to $J^{(1)}_{i,i+1}$ *(iii)* creation of third neighbour couplings out of second neighbour couplings can be neglected. As above, we define the distribution $\mathcal{P}^{(2)}(\zeta,\Gamma) := \mathcal{P}^{(2+)}(\zeta,\Gamma) + \mathcal{P}^{(2-)}(\zeta,\Gamma)$ as the sum of "distributions" of positive and negative next nearest neighbour couplings. With the above hypothesis, its scaling behavior is found to be described by

$$\frac{\partial \mathcal{P}^{(2)}(\zeta)}{\partial \Gamma} = \frac{\partial \mathcal{P}^{(2)}(\zeta)}{\partial \zeta} - \mathcal{P}^{(2)}(\zeta)\left(2\mathcal{R}(0) + \mathcal{P}^{(1)}(0)\right) + 2\mathcal{R}(0)\int_0^\infty \mathrm{d}\zeta_1\, \mathrm{d}\zeta_2\, \mathcal{P}^{(1)}(\zeta_1)\mathcal{P}^{(2)}(\zeta_2)\delta(\zeta-\zeta_1-\zeta_2) + \mathcal{P}^{(1)}(0)\ \delta(\zeta-\Lambda) \quad (2)$$

where Λ is an arbitrary large constant which stands for the negligible J in log. coordinates, and is taken to ∞ at the end of calculations. The Γ dependance of the distribution has been omitted for clarity. With the above hypothesis, the probability distributions for fields and nearest neighbour couplings still follow the equations for the chain. Hence, at the "Spin Glass" critical point, we can insert the scaling form $\mathcal{R} = \mathcal{P}^{(1)} = \mathcal{P}^*$ in (2). It is usefull to split $\mathcal{P}^{(2)}(\zeta,\Gamma)$ into a Λ independent part $\mathcal{P}_i^{(2)}(z,\Gamma)$ and $\mathcal{P}_\Lambda^{(2)}(z,\Gamma)$. By denoting $p(z,\Gamma)$ the Laplace transform in ζ of $\mathcal{P}^{(2)}(\zeta,\Gamma)$, we find that for z and Γ finite and fixed, $p_\Lambda(z,\Gamma) \to 0$ when $\Lambda \to \infty$. Then we show that the norm of the two parts of the solution satisfy : $\|\mathcal{P}_i^{(2)}\|_\zeta = 1 - \|\mathcal{P}_\Lambda^{(2)}\|_\zeta = \lim_{z\to 0} p_i(z,\Gamma) = \Gamma_0/\Gamma$, corresponding to a constant "decrease" of the couplings $J^{(2)}$. In this regime, the system "forgets" its initial conditions and flows to a general state gouverned by $\mathcal{P}_\Lambda^{(2)}$. Consistency of conditions *(i)* to *(iii)* can also easily being checked from the properties of the Laplace transform. As a consequence, such small next nearest neighbor couplings correspond to an "irrelevant perturbation" at this infinite disorder fixed point.

To go beyond this perturbative analysis, we have studied the scaling behavior of the zig-zag ladder by implementing numerically the above renormalization rules. We start by choosing a random configuration of fields h_i and couplings $J^{(1)}_{i,i+1}, J^{(2)}_{i,i+2}$ according to the previous initial distributions probabilities. Then at each step, the energy scale is lowered and the number N of spins is reduced by 1 according to the decimations rules specified above. This process is continued up to the last remaining spin, and repeated for a number $R = 10^3$ configurations. No assumption is made on the topology of the renormalized lattice, and we keep *a priori* all generated couplings. However, for practical reasons it appears necessary to restrict ourselves to energies larger than a lower cut-off Ω_{min}. With this procedure, the distributions $\mathcal{P}(\zeta,\Gamma), \mathcal{R}(\beta,\Gamma)$ are correctly sampled below $\Gamma_{max} = \ln(\Omega_0/\Omega_{min})$ [7]. For most of our results, this cut-off Ω_{min} was maintained to negligible values, without any noticeable incidence on the results. For fixed $J^{(2)}_{max}$, the transition is reached by varying the maximum amplitude h_{max} of the fields. We locate a putative infinite disorder phase transition by using the analogy with percolation [14]. At each decimation step i, corresponding to a system size $N_0 - i$, we consider the number of realizations $n_h(i)$ where a field was decimated at step i,

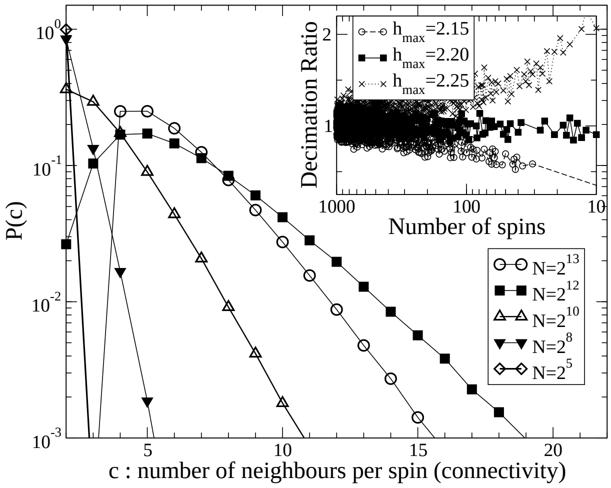

FIG. 1: Scaling behavior of the distribution of connectivity $P(c)$ for different number N of remaining clusters (spins), for the $J_1 - J_2$ model. The initial size is $N_0 = 2^{14} = 16384$ spins, and the decimation was performed over 1000 samples. After a transient regime characterized by an algebraic distribution of connectivity, the distribution ultimately renormalizes towards a delta function $c = 2$ corresponding to the topology of the chain. The inset show the ratio of number of decimated fields over the number of decimated bonds as a function of the number of remaining clusters N (see text). The critical point is evaluated as $h_{max} = 2.20 \pm 0.05$.

the similarly for the bonds $n_J(i)$. At the transition, the ratio $n_h(i)/n_J(i)$ should become scale invariant, whereas it should diverge or decrease to zero respectively in the disordered or ordered random phases. Moreover, the scaling behavior of this ratio is an excellent way to check for possible finite size effects respective to the topology of the initial lattice. The inset of the figure 1 shows this scaling of the decimation ratio for two values around the candidate critical value of h_{max}. Once such candidates for the transition are determined, we have studied the scaling behavior of the distributions functions $\mathcal{P}(\zeta,\Gamma), \mathcal{R}(\beta,\Gamma)$, of the distribution of magnetization $\mu(\Gamma)$, and number of active spins $n(\Gamma)$ in the clusters. This allows to characterize the criticality of the infinite disorder fixed point. Moreover, to fully characterize an infinite disorder fixed point beyond the simple chain, one should also be able to determine the renormalized topology of the critical lattice, and the associated correlations with the couplings. In a first attempt to study the scaling of this topology, we have followed the distribution of the connectivity of the lattice as the decimation goes on. The results, depicted on fig. 1, shows that while initially all sites have only 4 neighbors, the distribution $P(c)$ flows towards an intermediate algebraic distribution at intermediates sizes. While highly connected sites appear, we find by varying our lower cut-off Γ_{max} that rather strong correlations exist between the bonds connecting these sites. And while the decimation is pursued, the distribution narrow back towards a delta function peaked on $c = 2$, *i.e* the lattice is ultimately renormalized towards a chain. We thus find that for the $J_1 - J_2$ model, the infinite disorder fixed point is always given by the fixed point of the chain (see above and [5]), in agreement with previous results on the similar ferromagnetic two-leg ladder [8]

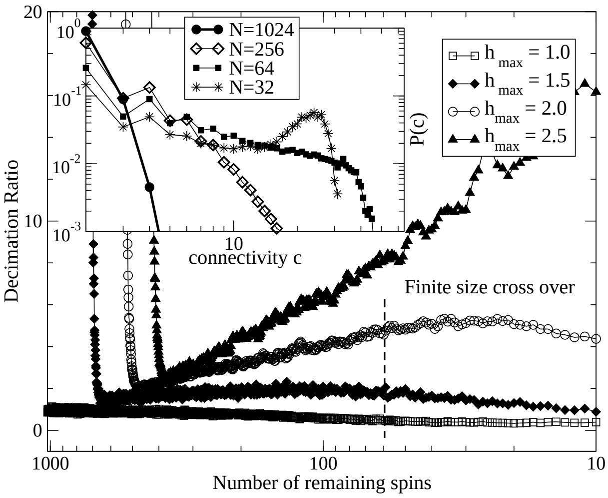

FIG. 2: Decimation ratio for the disordered quantum Ising model on a small world lattice. The initial size is $N_0 = 1024$, $q = 0.1$ and 5000 samples were used. The scaling behaviour of the distribution of connectivity for $h_{max} = 1.5$, in the inset, shows a broadening of this distribution up to some clear finite size topological effects inherent to small world models.

The previous results motivated the study of the opposite limit of long-range couplings competing with the initial couplings of the chain. Thus we naturally consider the hamiltonian (1) on a small world lattice[11], where beyond the previous nearest neighbors couplings $J^{(1)}_{i,i+1}$, we add random infinite range couplings $J^{LR}_{i,j}$ between any two non-neighbor sites i and j, with density p/N. In this paper, the existing couplings $J^{LR}_{i,j}$ and $J^{(1)}_{i,i+1}$ are distributed with the same uniform distribution between 0 ad 1. With these conventions, the average initial connectivity of this lattice is $2 + q$. Results of the same numerical decimation procedure as above indicate a phase transition different from the previous one (Zig-Zag ladder). In particular, contrarily to the previous case, the distribution of connectivity of the renormalized lattice broadens without limit up to some finite size effects. This cross-over happens when the numerical upper bound of the renormalized distribution $P(c)$ becomes of the order of the system size. Once this happens, highly connected sites proliferate, leading to a mean-field like behavior.

In this paper we have shown how the presence of random signs and further neighbor couplings affect the critical behavior of the random quantum Ising chain. We have particularly focused on the topological properties of the renormalized lattice, and we have explicitly shown how the presence of second neighbors couplings (Zig-Zag ladder) leads to an asymptotic lattice equivalent to a simple chain, proving the irrelevance of the second neighbor couplings perturbation at the infinite disorder fixed point of the chain. On the other hand, the results of our numerical renormalization approach show that the inclusion of an arbitrary density of long range couplings in the chain modifies the scaling behavior of the lattice's topology, and thus the associated critical behavior. These results stress the importance of determining the renormalized topological properties at any possible infinite disorder transition beyond the one-dimensional examples. In particular, the intermediate regime we have identified in our study of the Zig-Zag ladder opens the possibility of new infinite disorder scenarii for models with correlated long-range couplings. A natural extension of the present work would certainly focus on random algebraic interactions and the effect of the dimension, possibly relevant to the understanding of the dipolar glass $LiHo_xY_{1-x}F_4$ in a transverse field [2].

D. Carpentier and P. Pujol would like to acknowledge F. Leonforte for collaboration in a preliminary investigation related to the present work.

[1] M.A. Kastner *et al*, Rev. Mod. Phys **70**, 987 (1998).

[2] G.Aeppli, T.F.Rosenbaum *et al.*, Phys. Rev. Lett. **59**, 1969 (1987) G.Aeppli, T.F.Rosenbaum *et al.*, Nature vol. 425, Sept. 2003.

[3] S. Ludwig and D. D. Osheroff Phys. Rev. Lett. 91, 105501 (2003)

[4] Y. M. Galperin, B. L. Altshuler, D. V. Shantsev, Proc. of NATO/Euresco Conf. "Fundamental Problems of Mesoscopic Physics: Interactions and Decoherence", Granada, Spain, Sept.2003

[5] D.S.Fisher, Phys. Rev. B **51**, 6411 (1995).

[6] C.Dasgupta and S.K.Ma, Phys. Rev. B **22**, 1305 (1980)

[7] O. Motrunich, S. C. Mau, D. Huse and D.S.Fisher, Phys. Rev. B **61**, 1160 (2000).

[8] Y.-C. Lin, N. Kawashima, F. Igloi, H. Rieger, Prog. Theor. Phys. Suppl. **138**, 479 (2000).

[9] H.Rieger and A.P.Young, Phys. Rev. B **54**, 3328 (1996); M. Guo, R. N. Bhatt and D. A. Huse, Phys. Rev. B **54**, 3336 (1996).

[10] L. Viana and A. J. Bray, J. Phys. C: Solid State Phys. **18**, 3037 (1985).

[11] R. Albert and A.-L. Barabasi, Rev. Mod. Phys. **74**, 47 (2002).

[12] T. Nikoletopoulos *et al.*, J. Phys. A: Math. Gen **37**, 6455 (2004); B. Wemmenhove, T. Nikoletopoulos, and J. P. L. Hatchett, (cond-mat/0405563).

[13] By using the standard Ising duality relations, we can reformulate this model as a random XY chain in a random field. However, as opposed to C. Doty and D.S. Fisher, Phys. Rev. **B** 45, 2167 (1992), where no criticality of the infinite disorder kind was found, we will focus on the regime of strong disorder in all variables.

[14] C. Monthus, O. Golinelli and Th. Jolicoeur, Phys. Rev. Lett. **79**, 3254 (1997).